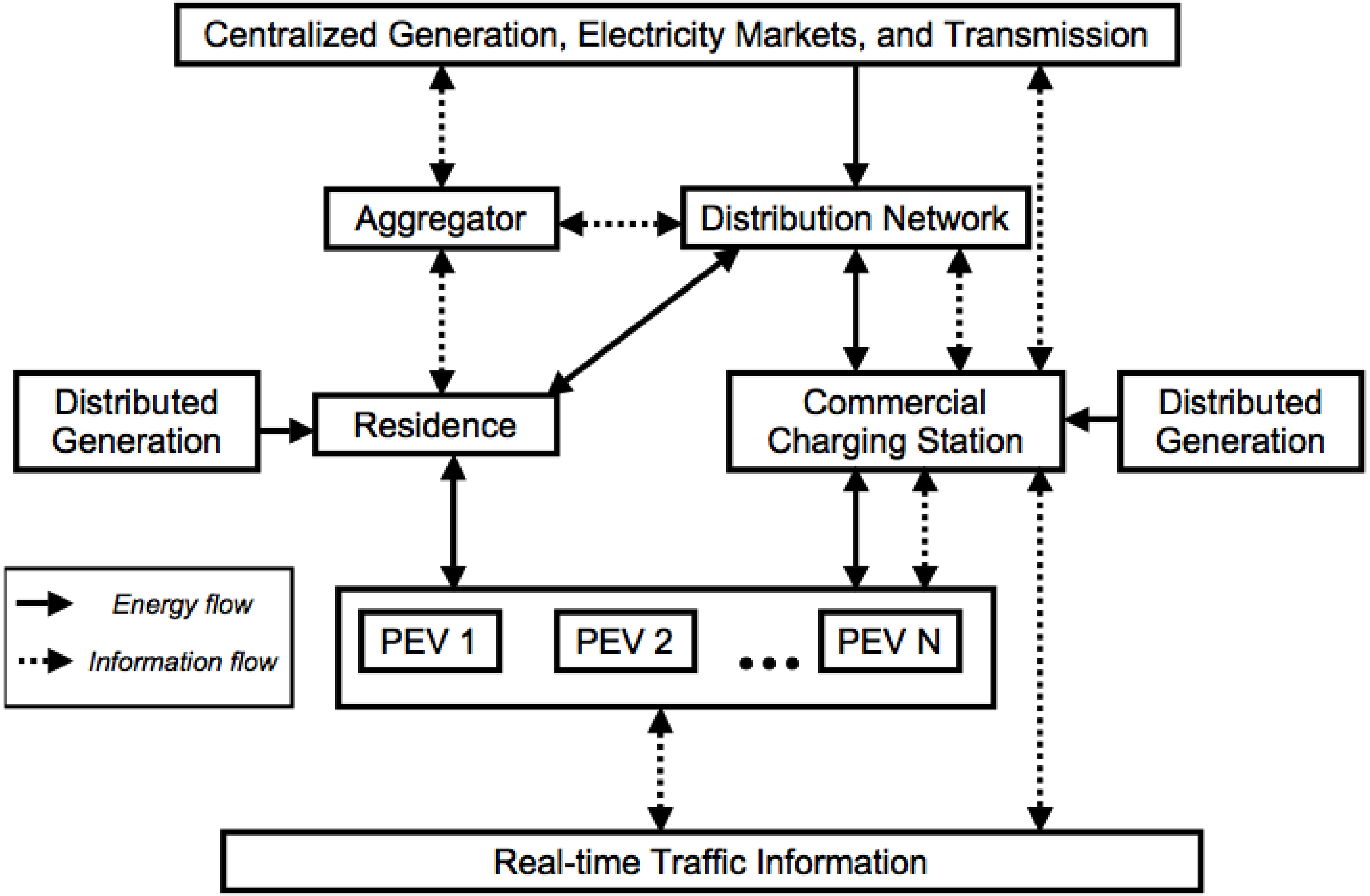
On Distributed Charging Control of PEVs with Power Network Constraints

Wann-Jiun Ma, Ufuk Topcu and Vijay Gupta

GLS 2013 on Smart Grid and the New Energy Economy

PEV Ecosystem



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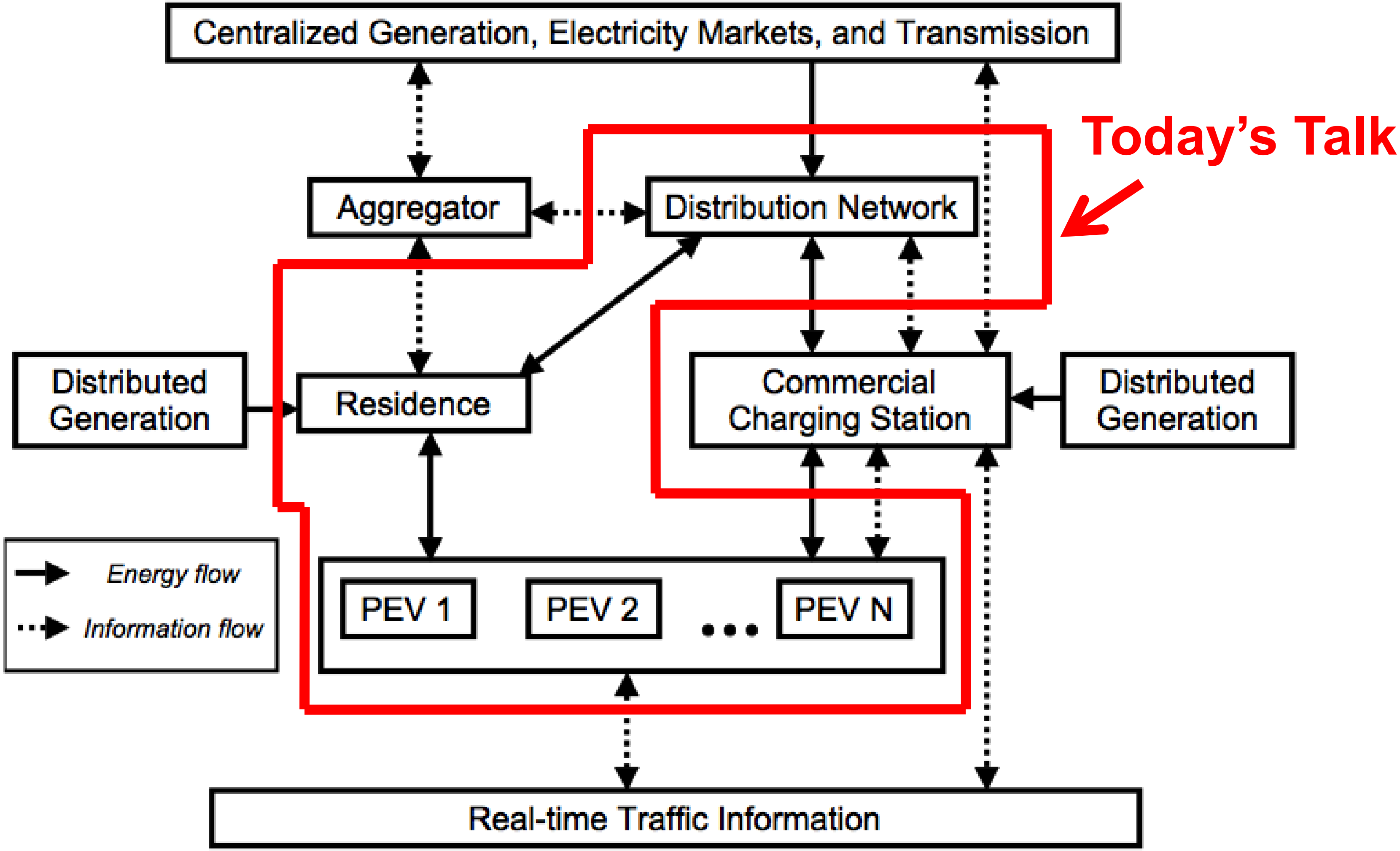
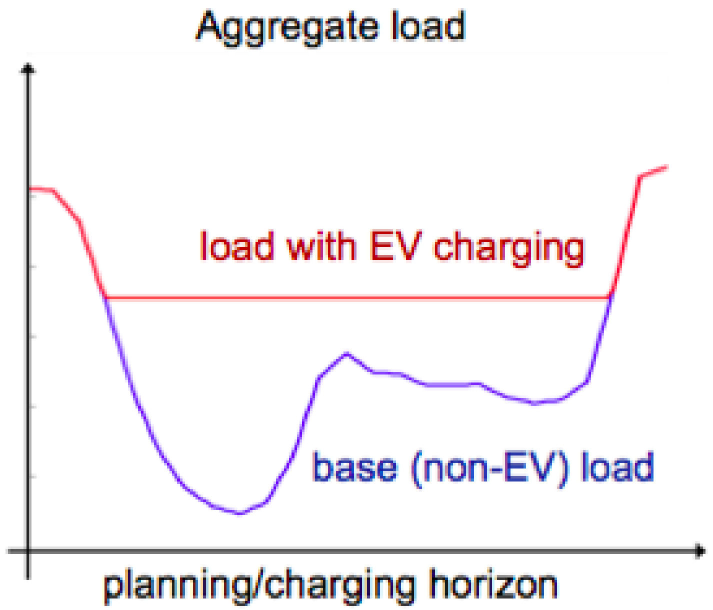


- Distributed charging of PEVs: capacity constraints, lack of communication infrastructure, privacy and security

Main Contributions

• Many distributed algorithms (based on pricing) have been proposed for inducing favorable charging profiles



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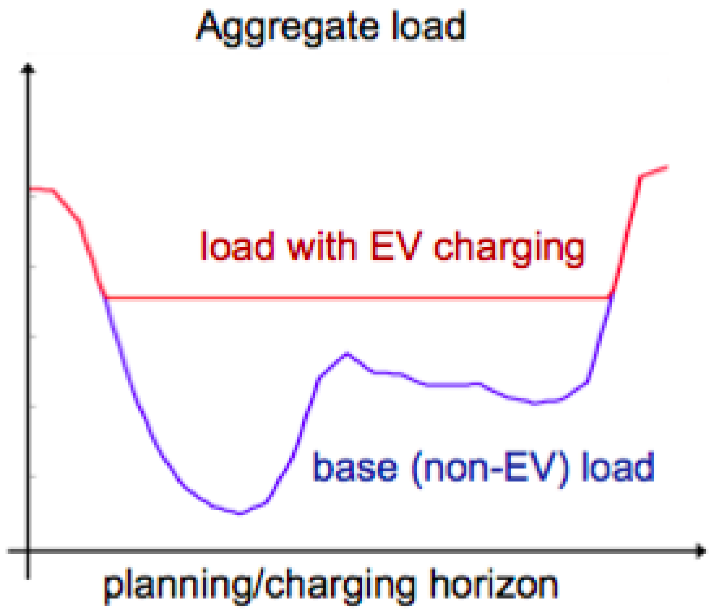

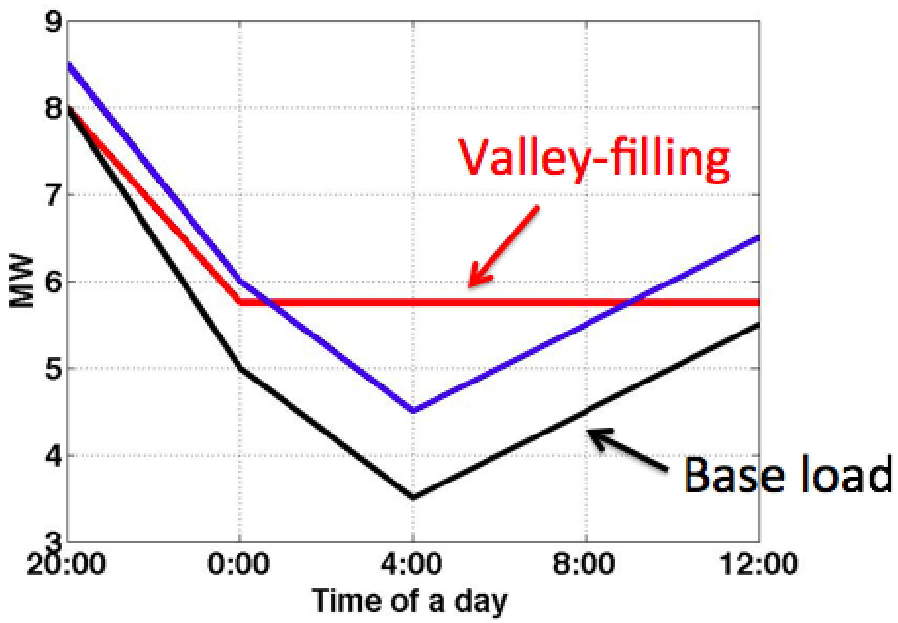

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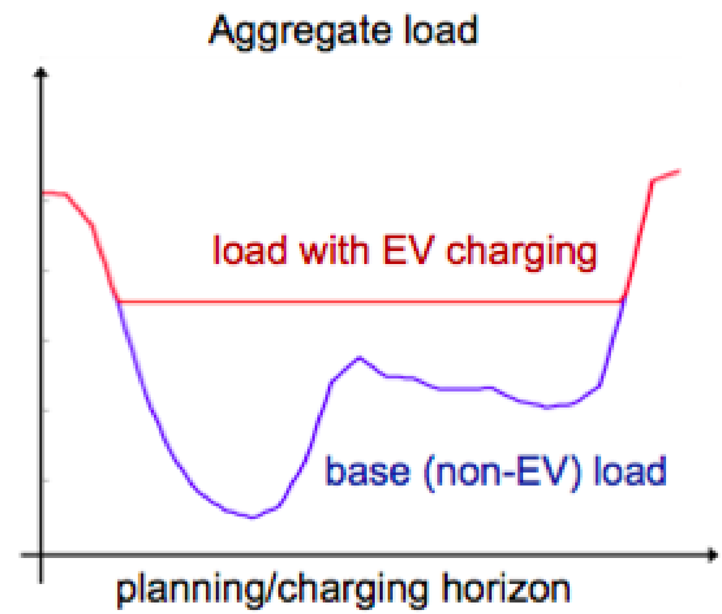

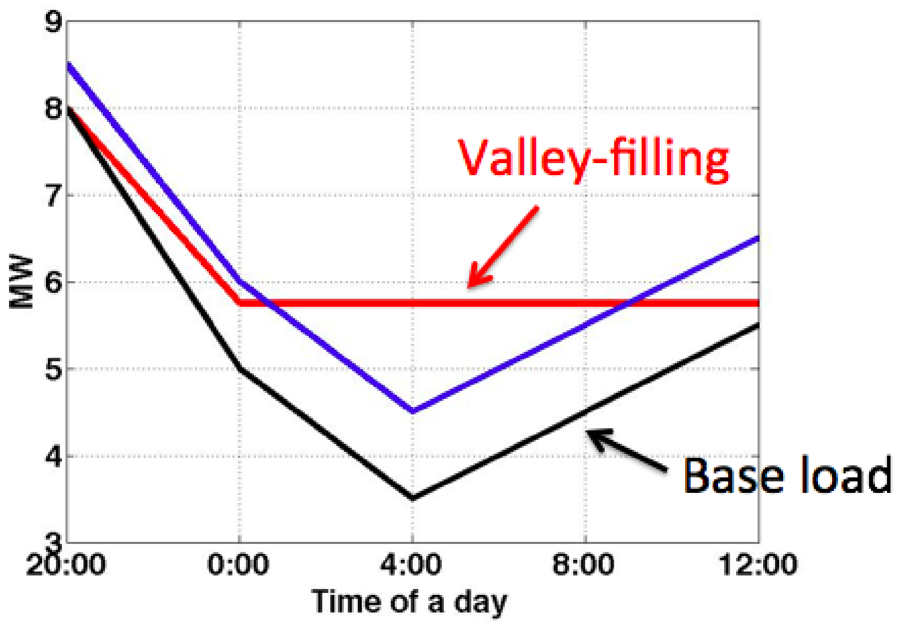

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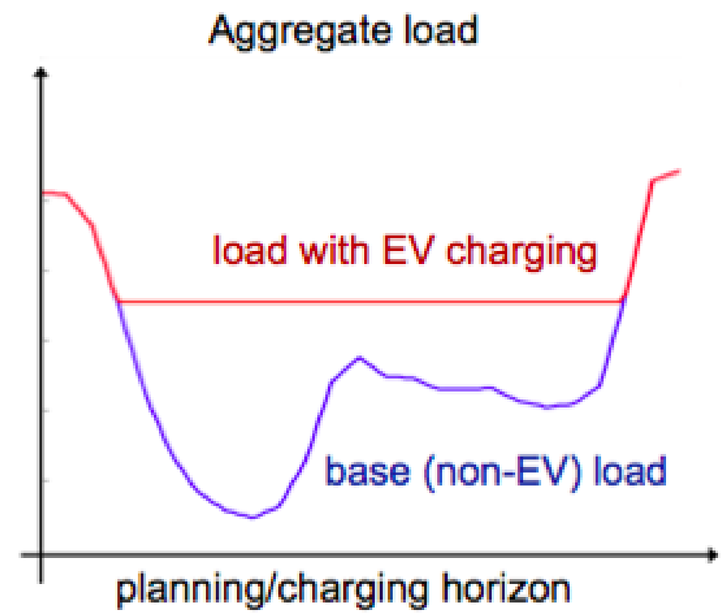

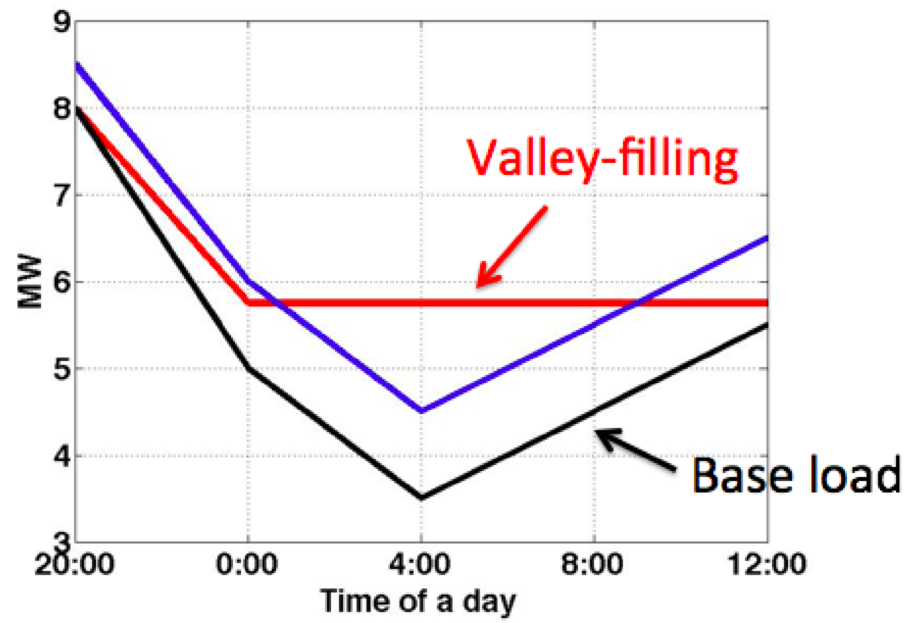

Basic Optimization Problem (From Gan et al, 2011)

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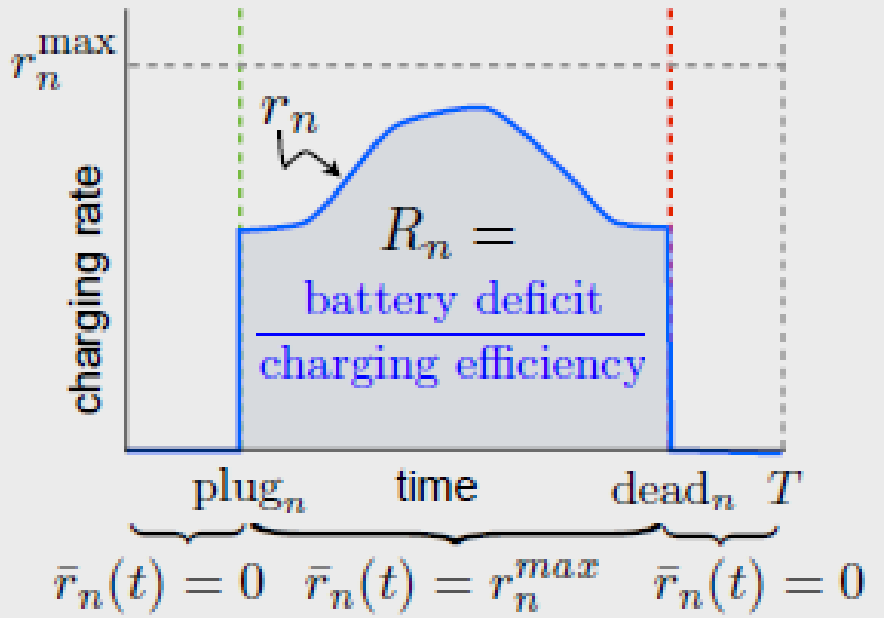
- Partition the planning horizon into T slots
- $r_n(t)$: charging rate of EV-n in time slot t



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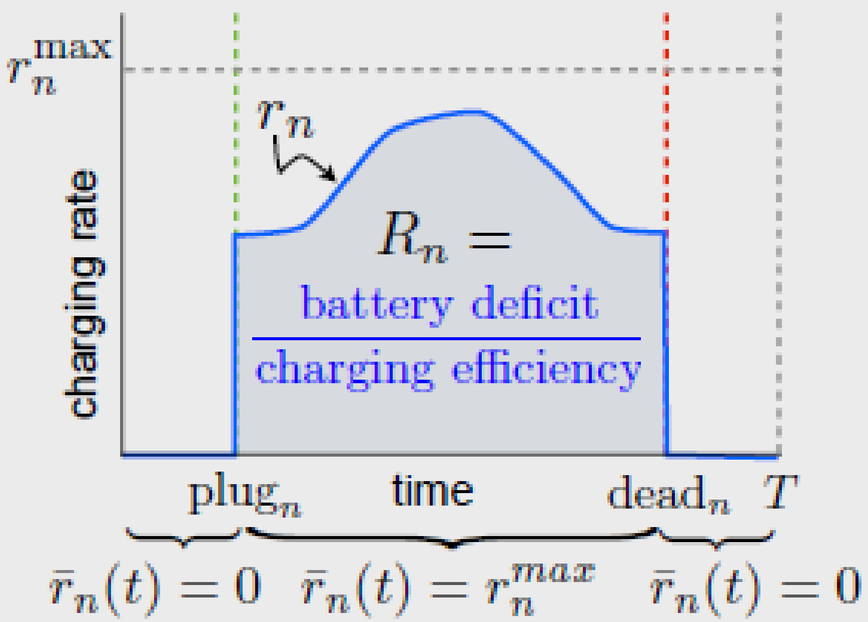


base load

$$\min \quad \sum_{t=1}^{T} \left(D(t) + \sum_{n=1}^{N} r_n(t) \right)^2$$

$$\text{s.t.} \quad 0 \le r_n(t) \le \bar{r}_n(t)$$

rate constraint

$$\sum_{t=0}^{T} r_n(t) = R_n$$

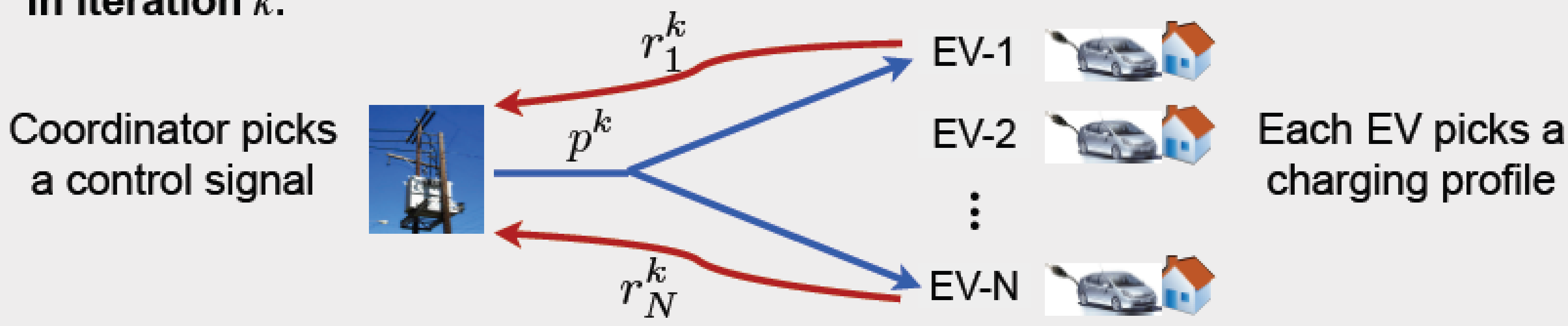
capacity constraint

over the variables

$r_n(t)$ for $t = 1, \ldots, T, \; n = 1, \ldots, N$

Basic Algorithm (Gan et al, 2011)

In iteration k:



$$p^k = \frac{1}{N}\left(D + \sum_{n=1}^{N} r_n^{k-1}\right)$$

e.g., price of electricity over the planning horizon

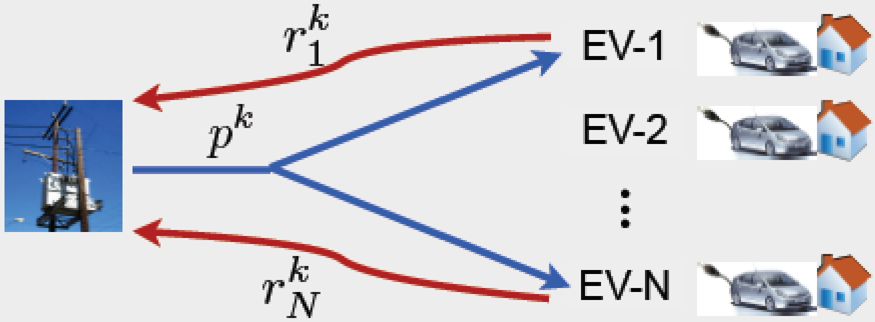
$$r_n^k = \underset{r_n \text{ feasible}}{\operatorname{argmin}} \; (\text{electricity cost})_n + \boxed{\frac{1}{2}\sum_{t=1}^{T}\left(r_n(t) - r_n^{k-1}(t)\right)^2}$$

deviation from the charging profile in the previous iteration

Basic Algorithm (Gan et al, 2011)

In iteration k:

Coordinator picks a control signal



Each EV picks a charging profile

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Theorem: Charging profiles converge to a profile in the set of optimal charging profiles.

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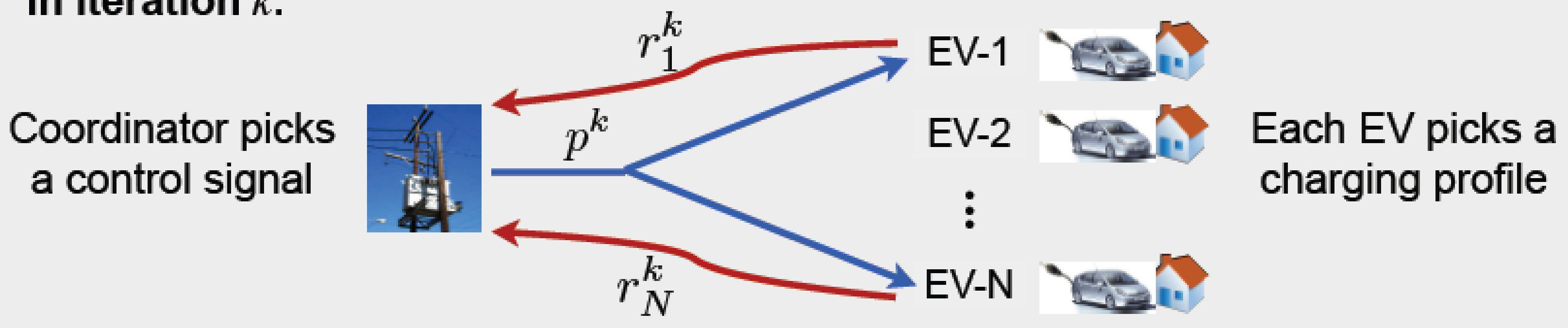

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Basic Idea for Including Active Power Flow Constraints

- Optimization problem

$$\min \sum_{t \in \mathrm{T}} \left(D(t) + \sum_{i \in \mathrm{N}} x_i(t)\right)^2$$

$$\text{subject to } 0 \le x_i(t) \le \overline{x}_i(t), \quad t \in \mathrm{T}, i \in \mathrm{N}$$

$$\sum_{t \in \mathrm{T}} x_i(t) = R_i, \quad i \in \mathrm{N}$$

$$Lx \le c$$

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Gan et al's formulation

Active power flow constraints

- Power flow constraints combine power flowing through various links and components
- All the terms in the constraint can be functions of time
- Assume that mapping temperature and life considerations to allowable capacity profiles has been done

Distributed Algorithms to Solve the Problem

- The algorithm from Gan et al needs to be augmented
- There needs to be another ‘price’ that charges for capacity constraint violations and is determined by negotiations with neighbors only
- We choose the method of alternating direction method of multipliers (ADMM) for setting this second price

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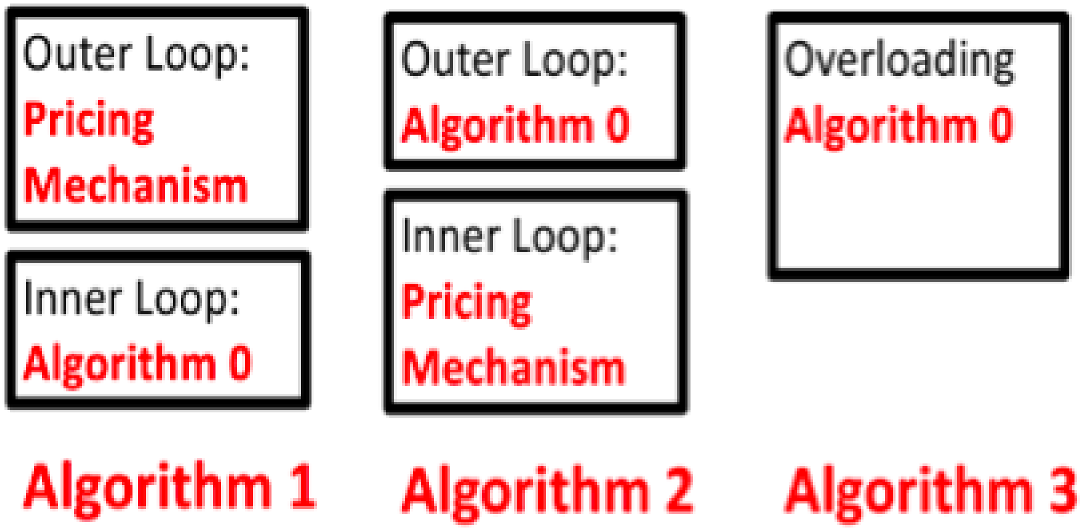

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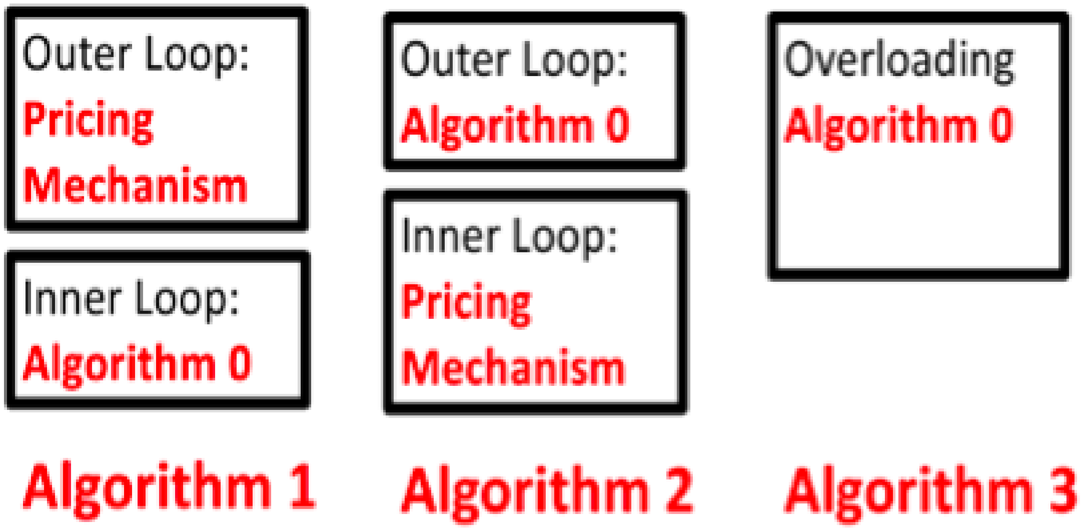
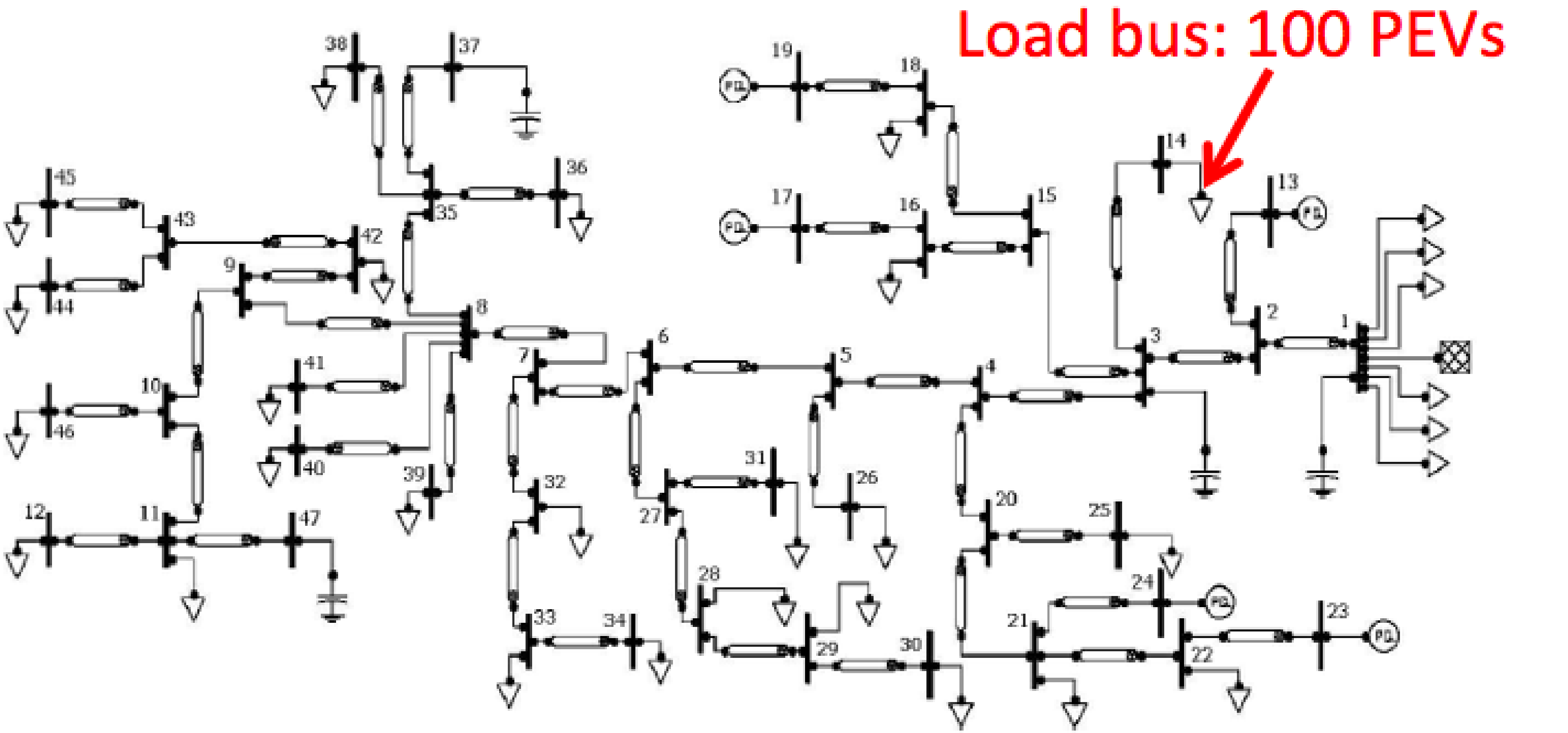
- Based on the time scales at which these prices are updated, different algorithms can be designed
- Hierarchical implementation is possible



Simulation Results

- Southern California Edison (SCE) 47 bus network with 100 PEVs at every load bus



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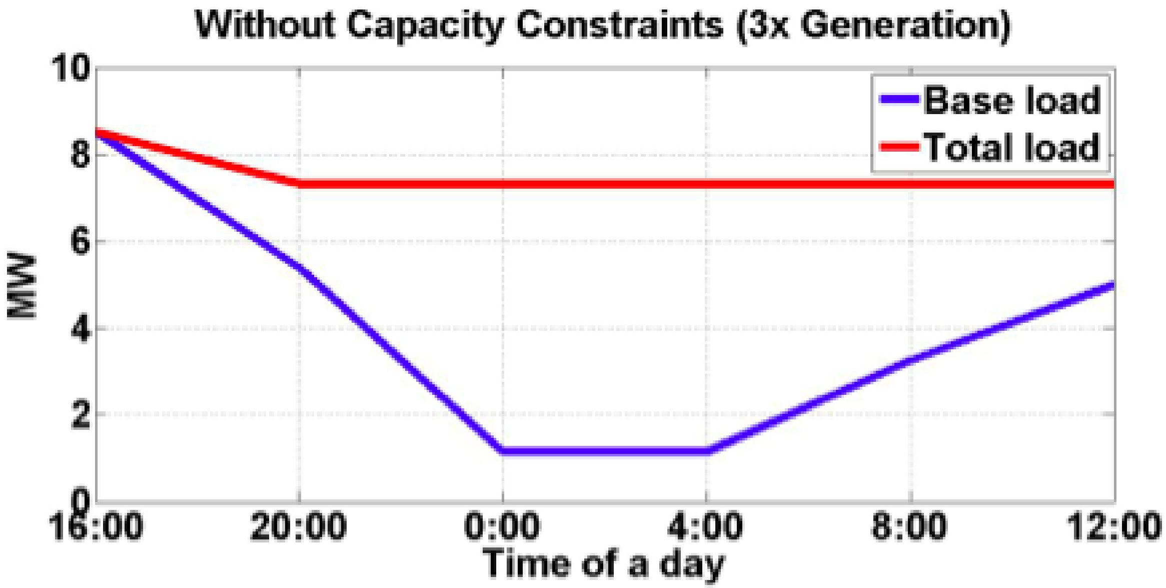

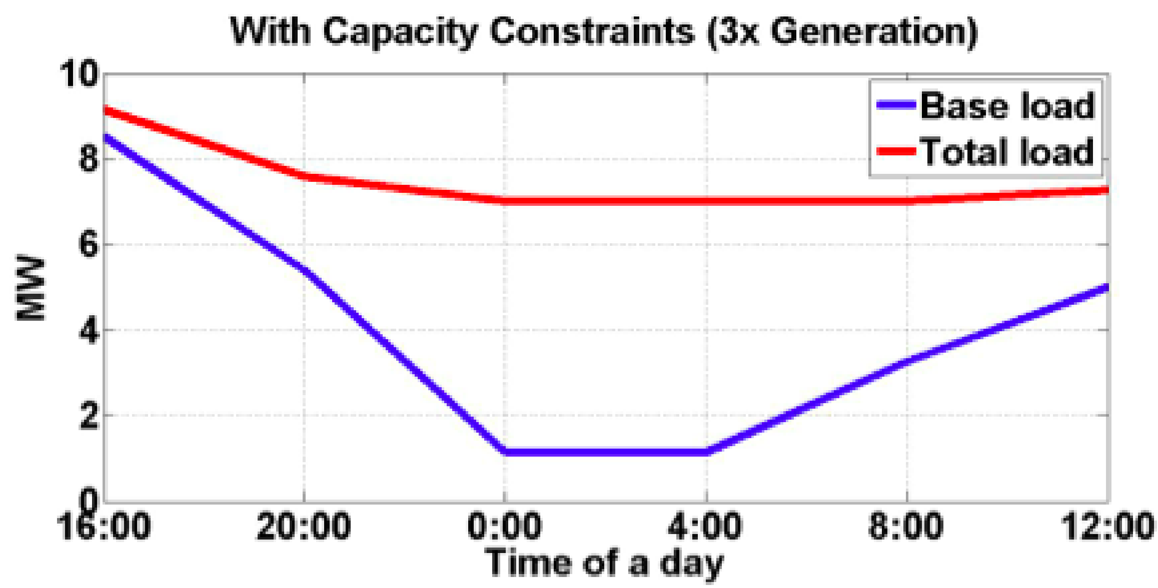

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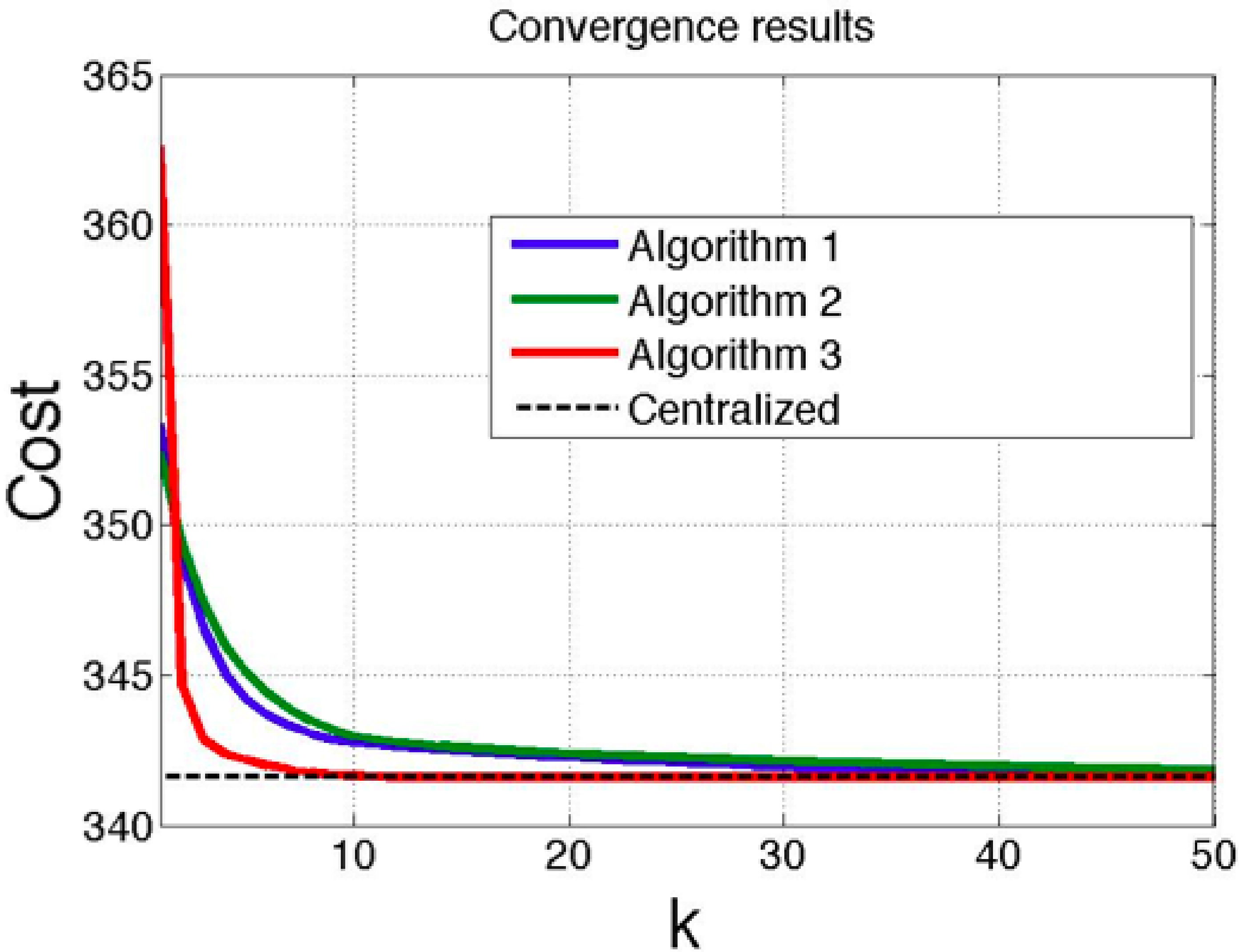

The frequencies that PEVs communicate with the utility company

Reactive Power Flow Constraints

- Need to add constraints on reactive power flow

$$p_j = P_{ij} - \bar{r}_{ij} l_{ij} - \sum_{k:(j,k)\in \mathrm{E},\, k\in \mathrm{M}} P_{jk}, \quad i,\ j \in \mathrm{M}\ ,\ (i,j) \in \mathrm{E},$$

$$q_j = Q_{ij} - \tilde{r}_{ij} l_{ij} - \sum_{k:(j,k)\in \mathrm{E},\, k\in \mathrm{M}} Q_{jk}, \quad i,\ j \in \mathrm{M}\ ,\ (i,j) \in \mathrm{E},$$

$$v_j = v_i - 2(\bar{r}_{ij} P_{ij} + \tilde{r}_{ij} Q_{ij}) + (\bar{r}_{ij}^2 + \tilde{r}_{ij}^2) l_{ij}, \quad i,\ j \in \mathrm{M}\ ,\ (i,j) \in \mathrm{E},$$

$$l_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_i}, \quad i,\ j \in \mathrm{M}\ ,\ (i,j) \in \mathrm{E},$$

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breaks convexity

Two relaxations

- Replace current constraint by inequality

$$l_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_i} \Rightarrow l_{ij} \leq \frac{P_{ij}^2 + Q_{ij}^2}{v_i}$$

- Can show that this relaxation does not introduce conservatism in the result

- Also need a regularization term in the cost function
- Interpretation of considering energy loss due to flow in the lines, but we are no longer performing valley filling
- However, the problem is now convex and can be solved with similar distributed algorithms as before

Difficulties in Multiple Rounds of Negotiation

- The optimization based algorithms may have privacy and security concerns since functions of charging profiles for the entire night are transmitted
- For the utility company, price profiles may carry too much proprietary information (esp. if valley filling is replaced by a direct dollar value objective)
- Communication infrastructure may not be able to support such multiple rounds of negotiation
- Can we reduce the amount of communication and make variables transmitted less informative?
- We propose the online learning based approach from game theory of regret minimization

Regret Minimization

- Consider the game of rock-paper-scissors

- Mixed strategies? Model the opponent?

Regret Minimization

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- Mixed strategies? Model the opponent?

- Learning based approach:
- Every time we play, observe the payoff and the opponent's action
- Compute regret
- Update action for next game to minimize regret
- Iterate

- Generic result: the strategy converges such that average regret goes to zero. Also if both players use regret minimization, then the strategies converge to Nash equilibrium

Regret Minimization for Distributed Charging

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- Payoff for PEVs is the price paid for charging

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- Payoff for utility company is variance of total load

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- Utility company can observe the total load and it transmits the price profile incurred in the morning
- Player choose action over next night to minimize regret (use gradient projection algorithm)

Regret Minimization for Distributed Charging

Customer Cost:

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Electricity Price

Utility Company Cost:

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Unknown and Uncertain Base Load

Regret:

k: Index for different days

$$R_i := \sum_{k=1}^{K} c_i^k(x^k) - \min_{x\in F_i} \sum_{k=1}^{K} c_i^k(x, x_{-i}^k)$$

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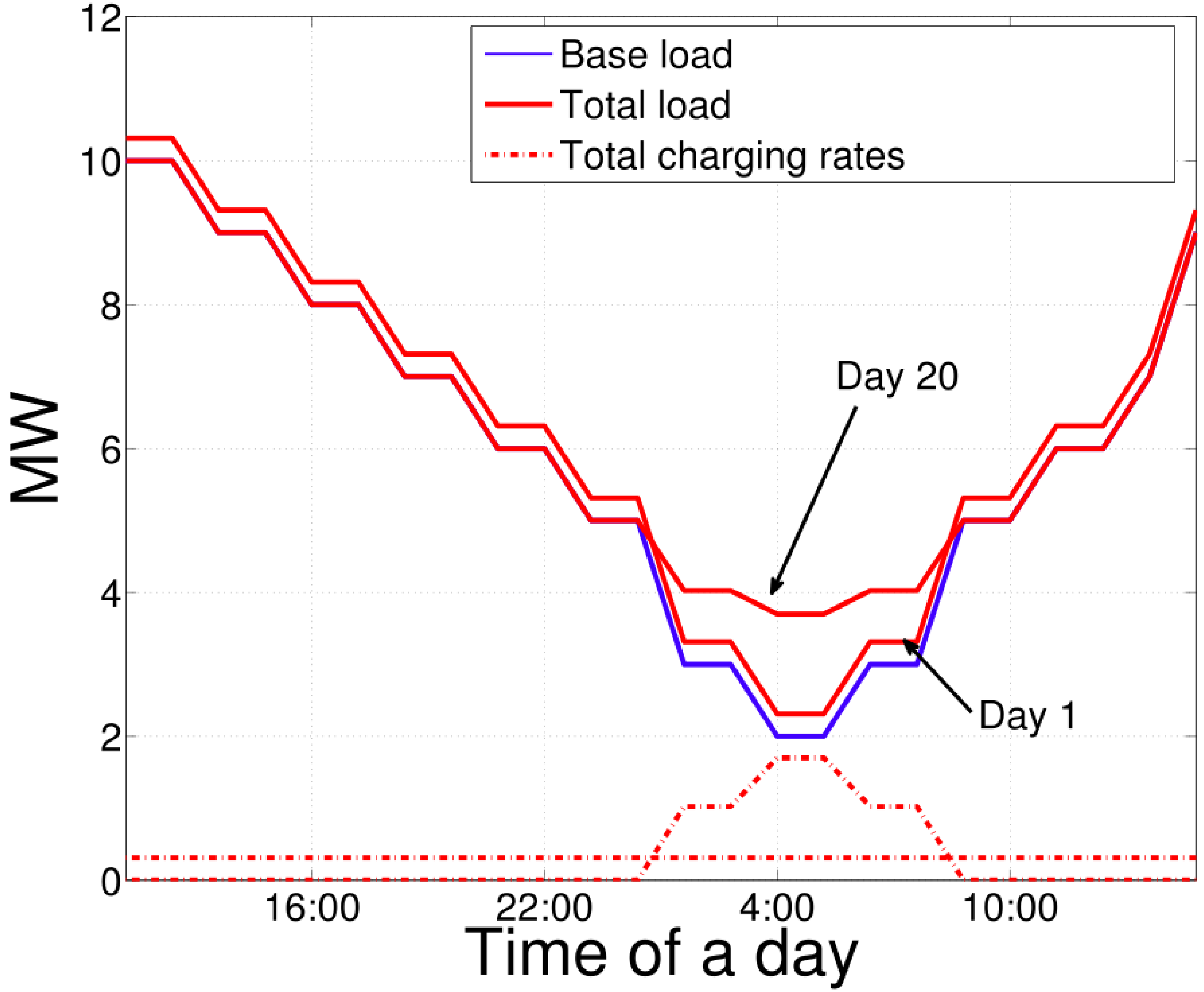
Use gradient projection to minimize both customer and utility company regret

$$x_i^{k+1} = \mathbf{Proj}\left(x_i^k - \eta_i^k \nabla c_i^k(x_i^k)\right)$$

Update only once at the end of the day

Simulation Example



Preliminary Analysis

- **Theorem**: Average regret converges to zero at least as fast as $O(\sqrt{K})$

  i.e. the regret is bounded by

$$R_i \leq \frac{||F||^2\sqrt{K}}{2} + \left(\sqrt{K} - 1/2\right)||\nabla c_i||^2$$

K ：Total days

$||F||$ ：The bound of the feasible set

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- **Theorem**: The average charging profiles and the pricing policy converge to a Nash equilibrium

i.e. the average charging profile $\bar{x}^K := 1/K \sum_{\tau=1}^{K} x^\tau$ satisfies

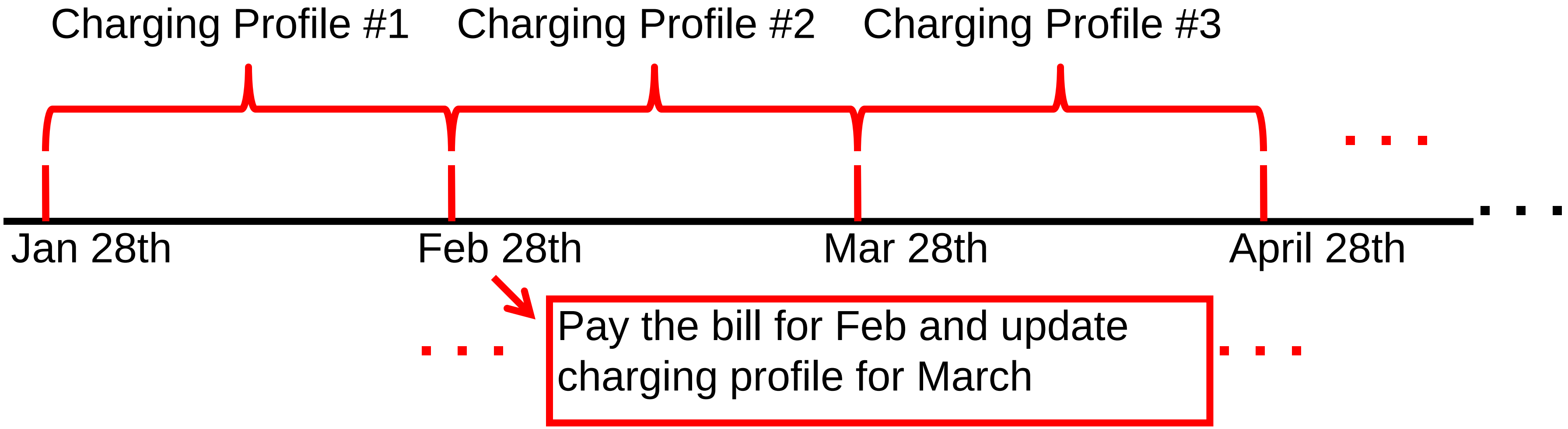
$$c_i(\bar{x}^K, x_{-i}^K) \leq c_i(y_i^K, x_{-i}^K), \quad K \to \infty$$

- Communication requirements are decreased and security and privacy concerns are met better. Only the incurred price profile needs to be published.

Extensions

- Real-time pricing may not be available every day
- Customers use the same charging profiles for the entire month; pay electricity bills at the end of month and update their charging profiles for the next month
- Does past information (e.g., past base load, past charging profiles) help to minimize the regret?
- Minimizing the regret adaptively

Example:



Conclusions: Distributed PEV Charging

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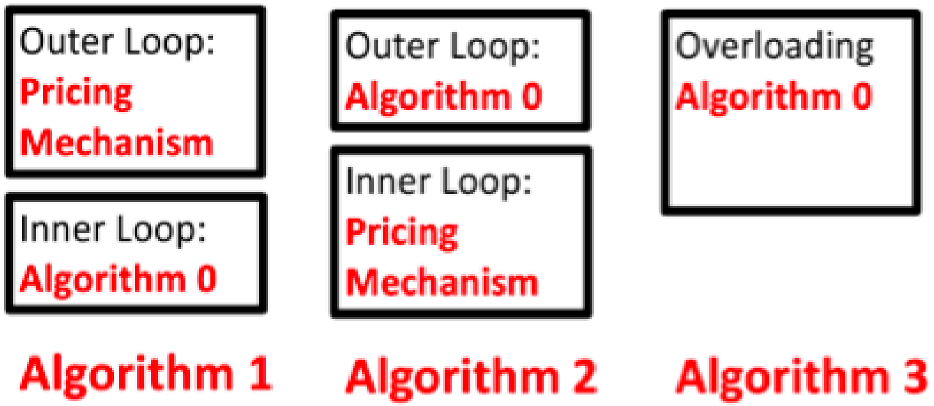

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